Teasers

For your recreational enjoyment here are a few teasers.

1. [Thanks to Andreas Tollaksen] Ten workers are producing 10-gram gold coins at the Mint. One of them is suspected of shaving a gram of gold from each coin for himself. How can the Mint discover who is the miscreant by a single weighing of coins?

   Answer: Take one coin from the first worker, two from the second, three from the third, etc. Weigh all 55 coins. The number of grams under 550 identifies the culprit. Incidentally, note that there are five pairings of coins totaling 11 grams — 1 plus 10, 2 plus 9, etc. In general the sum of the first n integers is $(n + 1)n/2$, easily proved by induction.

2. At what time exactly after 1:00 is the minute hand of a clock on top of the hour hand?

   Answer: Most people approach this problem as a convergent infinite series, where the terms are the times it takes for the minute hand to catch up to where the hour hand had been — easy enough, but not the most elegant. Consider that both hands of a clock move at constant angular velocity, with the minute hand arriving over the hour hand exactly 11 times in a 12 hour period. Thus the time after 12:00 to the next conjunction is 12/11 hours. The time then is $1{:}05{:}27\frac{3}{11}$.

3. How can you determine the thickness, without measuring, of a roll of kitchen paper, knowing only the length of the roll and the inside and outside radii of the tube on which the paper is rolled?

   Answer: Consider the volume of the paper. Let l be the length, w be the width, and t be the thickness. Then the volume is lwt. Now let r and R be the inside and outside radii of the tube with paper. The volume, therefore, is also $\pi(R - r)^2 w$.
   Equating these two expressions — noting that the width neatly cancels — and solving for t gives the solution: $t = \pi(R - r)^2 w/l$. Note that the equation could also be written directly without reference to the width, which is an irrelevant quantity, but the 2-dimensional analysis is somewhat less intuitive.

4. In bowling how many different possible scores are there at the end of the fifth frame on the way to a 200 game? (You may assume, correctly, that a maximum score in any of the 10 frames is 30, and that a minimum score is zero, with all possible scores in between.)

   Answer: One can score no more than 150 in either half of the game, so one must score at least 50 in the other. The combinations are 150 and 50, 151 and 49, 152 and 48, etc., of which there are 101 such combinations.