

## Reciprocal Ownership

*Question:* Assume Company A owns fraction  $x$  of Company B, whereas Company B owns fraction  $y$  of Company A. What is Company A's beneficial interest in Company B, and reciprocally, what is Company B's beneficial interest in Company A?

*Answer:* The first slice of Company B owned by Company A is the fraction  $x$ . The second slice is the fraction  $xyx$  coming back through a loop to A, then to B again. Continuing, the third slice is the fraction  $xyxyx$ . The pattern continues indefinitely producing the fraction  $x^{i+1}y^i$  after the  $i^{\text{th}}$  loop. The sum of these fractions, the beneficial interest  $I_{ab}$  of A in B, is

$$I_{ab} = x \sum_{i=0}^{\infty} x^i y^i = \frac{x}{1 - xy}$$

With similar reasoning the beneficial interest  $I_{ba}$  of B in A is

$$I_{ba} = y \sum_{i=0}^{\infty} y^i x^i = \frac{y}{1 - yx}$$

*Example:* Company A owns 20% of Company B, whereas Company B owns 30% of Company A.

$$I_{ab} = 0.20 \sum_{i=0}^{\infty} 0.20^i 0.30^i = \frac{0.20}{1 - 0.20 \cdot 0.30} = 21.277\%$$

and

$$I_{ba} = 0.30 \sum_{i=0}^{\infty} 0.30^i 0.20^i = \frac{0.30}{1 - 0.30 \cdot 0.20} = 31.915\%$$