Polygon Edges

A polygon of n sides has an area of n^2. As n grows large without bound the length of a side of the polygon approaches what limit?

Solution

Observe that a polygon with a large number of edges is well approximated by a circle. So, first determine the radius of a circle with the same area as the polygon. From there compute the circumference. This value approximates the sum of all sides of the polygon, of which the number is known to be n. The result follows.

To proceed, then, observe that the radius r of the approximating circle is $n/\sqrt{\pi}$. The circumference, therefore, is $2\sqrt{\pi}n$. But this value approximates the perimeter of the polygon, which is ln, with l the length of a side. This length, therefore, is $2\sqrt{\pi} \approx 3.544908$.

Note that this approximation is not bad, even for low values of n. For a square, with an area of 16 the side is obviously 4, high compared to l by 12.8%. For a hexagon, with an area of 36, the side is

$$\frac{6}{\sqrt{\frac{3}{2}\sqrt{3}}} \approx 3.722419,$$

which is high compared to l by 5.0%. For an octagon, with an area of 64, the side is

$$\frac{8}{\sqrt{2(\sqrt{2}+1)}} \approx 3.640719,$$

which is high compared to l by 2.7%. Note that one may derive these formulas for the hexagon and octagon easily by resort to triangulation. This is a good exercise. It may help to note that the area of a "slice of the pie" on a polygon is n.