

Solution to “Paying the lawyers”

“If you have ever received an outrageous bill from a law firm (and who in business hasn’t?) and were wondering where all those hours went, well contemplate this.

“Your friendly team at Pleadem & Speedem has decided it is your best interest that the three lawyers on your case work as follows. Each studies your matter alone in his office for one hour. So, you get billed for those three hours. Then each pair of lawyers, of the three possible pairs, studies your matter in conference for an hour. Bill another six hours. Then, of course, all three have to meet in conference for an hour. Bill a final three hours. The total is 12 hours, or *four times* as many hours than if the three lawyers had just studied your matter alone for an hour, or equivalently, had the three only met all together for one hour.

“Now, your assignment is to produce a formula for this multiple – in the given example, *four times* – for an arbitrary number of lawyers. As a corollary, give the multiple in the instance of 10 lawyers.”

This is a straightforward combinatorial problem based on binomial coefficients. The total number of hours billed, where the terms in sequence refer to the groups of 1, 2, 3, etc., lawyers, is

$$\sum_{k=1}^n k \cdot \binom{n}{k} = n \cdot \sum_{k=0}^{n-1} \binom{n-1}{k} = n \cdot 2^{n-1}$$

So, compared to the reference number of hours billed for n lawyers working alone, or in one group of n , the final bill is 2^{n-1} times as much. Therefore, for 10 lawyers working on your case this multiple is 512, or 5120 total hours compared to 10 hours.

Now you know why your bill is so big.