BI Norwegian School of Management

GRA 6535 Derivatives

Midterm Examination, with Answers

16 February 2010

Please answer all six questions.

1. A financial institution wishes to hedge over the short term a portfolio of mortgages. This portfolio has duration of –11 and a market value of $9 million. Looking to the nearby U.S. Treasury bond futures market the portfolio's manager observes a quotation of 80 (percent of par) and identifies the cheapest-to-deliver cash bond as having duration of –20. If the price to yield 6% on this bond, with semi-annual compounding for each $1 of par is 1.2500, and if the pertinent discount factor for the interval of time to delivery is 0.99, what is the cash price of the cheapest-to-deliver bond assuming no arbitrage opportunities exist? How many contracts of the $100,000 nominal par bond futures are required for the hedge? Does the financial institution go long or short?

This idea here is to compare durations of the portfolio to be hedged and the cheapest-to-deliver bond, and also the amounts involved, reduced to present values. All, except the futures price of the cheapest-to-deliver bond are in present value. To convert that amount simply multiply the futures price by the delivery factor, and again by the nominal par value of the contract, and discount. The resulting number of contracts to go short, because this is a hedge of an asset on the books, is

$$\frac{-11}{-20}\frac{\$9.0}{(0.80)(1.2500)(\$0.1)(0.99)} = 50$$

2. The spot price for one British pound in dollars is $2.00. The three-month forward rate for British pounds is $1.90. If 4.5% is the prevailing interest rate over this period in dollars, what is the interest rate in British pounds providing parity?

In parity an investor has no advantage either in dollars or pounds. Hence the following amounts must be equal, substituting r *for the unknown.*

$$(1 + r)(1.9000) = (1.045)(2.0000)$$

Thus r = 0.10, *or* 10%.

Note: Full credit is also given for the factor 1.01125 *instead of* 1.045, *leading to* r = 25.79% *annualized, as the question was somewhat ambiguous about whether the dollar rate was quarterly or annual. For simplicity it was assumed quarterly.*

3. A bond priced at 110 has duration of –8 and convexity of 40. If the yield declines by 1% what is your estimate of the new price of the bond? (A result to two decimal places is all right. Do not bother to convert to 32^{nds}.)

Recall Taylor's series for the ratio of a bond price V *as a function of yield* r, *to its value* V_0 *at a specific yield* r_0.

$$V(r) = V(r_0) + V'(r_0)(r - r_0) + \frac{1}{2}V''(r_0)(r - r_0)^2 + \cdots$$

Dividing by $V(r_0)$ *gives*

$$\frac{V(r)}{V(r_0)} = 1 + \frac{V'(r_0)}{V(r_0)}(r - r_0) + \frac{1}{2}\frac{V''(r_0)}{V(r_0)}(r - r_0)^2 + \cdots$$

Substituting the known quantities provides

$$\frac{V(r)}{110} = 1 + (-8)(-0.01) + \frac{1}{2}(40)(-0.01)^2 + \cdots,$$

or $V(r) = 110 * (1.0820) = 119.02.$

4. In March a bank anticipates issuing $11.7 million of zero coupon certificates of deposit with two-year maturities in September, and wishes to hedge this position. The market of choice is U.S. Treasury notes. The September contract is trading at 80, whereas the cheapest-to-deliver note has duration of –5 and a delivery factor of 1.2000. Short-term interest on loans to arbitrageurs is available at 5% at discount, compounded semiannually (meaning that the discount factor is 0.975 for a six-month period.) What action does the firm take? When are the hedges placed? When are they removed?

Note: There were several misleading errors in the original statement of this problem, and as a consequence all students will receive full credit for their answers. The problem as stated here is a correctly revised version, which we shall discuss in class. Here is the answer to the revised problem.

The two year zero coupon CD's have duration -2, *which is compared to the duration of* -5 *for the hedge instrument. The analysis is similar to that of Problem 1. The number of contracts in this instance, also short, as this is a prospective liability, is*

$$\frac{-2}{-5}\frac{\$11.7}{(0.80)(1.2000)(\$0.1)(0.975)} = 50$$

The hedges are placed in March, when the decision is made to sell the CD's in September. The hedges are lifted when the sales are actually made in September.

5. The stock portfolio of a mutual fund worth $500 million has a beta value of 1.2. The manager wishes to hedge this position in the S&P 500 futures market. The spot S&P portfolio has a dividend yield of 3%, and short-term interest is 4%. How many of the 'big' S&P futures contracts (value of each: $500 times the index, now at 1000) does the manager choose? Are they long or short? What is the equilibrium futures price,

six months hence? (For simplicity assume that all interest, dividends, and mark-to-market cash flows on the futures contract come at the end of the six-month period.)

Compare the beta values to determine the hedge ratio. The beta value of the market, as represented by the S&P 500, is 1.0. One goes short for this asset on the books

$$\frac{1.2}{1.0}\frac{\$500{,}000{,}000}{\$500 * 1000} = 1200 \text{ contracts.}$$

As there is a 1% net premium of borrowing cost over dividend yield, the investor requires a higher price by the difference for this cash and carry play. The futures price for a contract scheduled for cash settlement in six months therefore is

$$1000\left[1 + \frac{.04 - .03}{2}\right] = 1005$$

6. A portfolio consists of two bonds. One has 80% of the value, and duration of –5. The portfolio duration is –8. What is the duration of the other bond?

Recalling the lemma that the weighted average of durations is the duration of the portfolio, using D for the unknown, one has

$$0.80 * -5 + 0.20 * D = -8,$$

whence it follows that $D = -20$, or simply ‘20’, if you choose.