BI Norwegian School of Management

GRA 6535 Derivatives

Final Examination

20 May 2010

The time for the examination is three hours. Please answer all six questions, which are weighted equally. Full credit will be given for an accurately displayed formula.

1. Consider the two-period binomial put option model on a share of stock. The price of the stock at present is 50, and the future values it can attain are 60.5, 49.5, and 40.5. The strike price is 55, and the periodic rate of interest is 5%. What is the expected future value of the put option at expiration? What is the expected future value of the companion call option?

   From the data, $u = 1.10$, $r = 1.05$, $d = 0.90$. *So* $p = 0.75$ *and* $1 - p = 0.25$. *Also,* $P_{uu} = 0.0$, $P_{ud} = 5.5$, $P_{dd} = 14.5$. *The expected future value of the put is therefore* $P_2 = p^2(0.0) + 2p(1-p)(5.5) + (1-p)^2(14.5) = 2.96875$. *The expected future value of the companion call, by parity, is* $C_2 = P_2 + Sr^2 - K = 3.09375$.

2. What is the lower bound of a one year European futures put option with a strike of 105.00 if the futures contract is trading at 94.20 with interest at 8%? What is the upper bound?

   The lower bound is zero, with the upper bound achieved when both the futures and the call are zero. By parity, this upper bound is $K\mathrm{e}^{-0.8} = 96.9272$.

3. A stock index is priced at 35. One year European call and put options with a strike of 42 are trading at 2 and 7, respectively, at parity. Short-term interest is 8.5%. What is the dividend rate on the index?

   The setup, by parity, is $C - P = S - K\mathrm{e}^{-(0.085-q)}$. *Substituting and solving for* q *yields* $q = 0.0362$, *or* 3.62%. *The alternative setup, favored by author Hull,* $C - P = S\mathrm{e}^{-q} - K\mathrm{e}^{-0.085}$ *is also acceptable.*

4. Show that *rho* for a European call (the sensitivity of call price to change in interest rate) is always greater than *rho* for the companion put, except at expiration, when the limiting difference is zero.
   Differentiating the parity equation with respect to r *yields* $\frac{\partial C}{\partial x} - \frac{\partial P}{\partial x} = Kr\mathrm{e}^{-rt} > 0$.

5. Assume the one-period binomial pricing model, with stock price S, having terminal values uS and dS. Let a call price be C, with corresponding terminal values C_u and C_d. Let r be the compounding factor (1 plus the interest rate.) Assuming risk-neutral

probabilities, what is the mean μ of the terminal stock price? the variance σ^2? Express this variance in terms of C and the other variables, thus providing the 'smile' function $\sigma^2(C)$.

One has by the pricing formula, $C = \frac{1}{r}[pC_u + (1-p)C_d]$*, whence it follows that* $p = \frac{rC - C_d}{C_u - C_d}$*, so* $1 - p = \frac{C_u - rC}{C_u - C_d}$*. It is easily determined that* $\mu = rS$*. Continuing,*

$$\sigma^2 = \frac{rC - C_d}{C_u - C_d}(uS - rS)^2 + \frac{C_u - rC}{C_u - C_d}(rS - dS)^2$$

$$= \frac{S^2}{C_u - C_d}[(rC - C_d)(u - r)^2 + (C_u - rC)(r - d)^2].$$

6. What arbitrage opportunity is available if a futures call is priced at 3, with its companion put at 6, if the strike is 70 and the futures are 67? What positions would you place to effect the arbitrage?

   By parity have $3 - 6 < e^{-rt}[67 - 70]$*. The call, therefore is relatively underpriced compared to the put. Do the reversal by buying the call, selling the put, and shorting the stock.*