Dynamic copula models for the spark spread

PAUL C. KETTLER

JOINT RESEARCH WITH
FRED ESPEN BENTH

PRESENTED TO THE
EUROPEAN REGION RESEARCH CONFERENCE

SOCIETY OF SIGMA XI
AALTO UNIVERSITY

24 MAY 2011

Motivation

This investigation focuses on

- the dependency between electricity and gas prices

Motivation

This investigation focuses on

- the dependency between electricity and gas prices
- option pricing for the spark spread

Motivation

This investigation focuses on

- the dependency between electricity and gas prices
- option pricing for the spark spread

The study builds on the fundamental work of Fred Espen Benth and Jūratė Šaltytė-Benth (2006).

The spark spread

The spread relationship

$$S(t) := E(t) - c\,G(t),$$

where $E(t)$ and $G(t)$, respectively, are electricity and gas prices quoted in customary units. The constant c is a "heat rate" chosen to make approximate equivalence between the energy content of the two sources.

The quotation conversion and heat rate

$$1\,\frac{\text{penny}}{\text{therm}}\cdot\left[\frac{1\text{ therm}}{105.5\text{ MJ}}\cdot\frac{1000\text{ MJ}}{\text{GJ}}\cdot\frac{3.6\text{ GJ}}{\text{MWh}}\cdot\frac{\$1}{100\text{ pence}}\right]$$
$$= 0.34123223\,\frac{\$}{\text{MWh}}$$

In this system of quotation the heat rate $c = 0.85308057$, now allowing for relative efficiency, gas to electricity, of 40%. This is the system of quotation and the heat rate used in the cited Benth and Šaltytė-Benth (2006) study.

The main difference between electricity and gas

Before all other considerations one must consider that

1. Gas is storable.
2. Electricity is not.

Well, electricity is somewhat storable, in indirect ways,
like water in reservoirs, or in finished products,
like bars of aluminum.

Preparation of the time series

The fundamental data underlying this study are daily prices for electricity and gas. These are first converted by log transform, then detrended and deseasonalized by the following linear and sinusoidal models, estimating parameters in each series.

$$\widetilde{E}_2(t) = a_E^{(1)} + a_E^{(2)} \widetilde{E}_1(t) + \epsilon_{E,1}$$
$$\widetilde{G}_2(t) = a_G^{(1)} + a_G^{(2)} \widetilde{G}_1(t) + \epsilon_{G,1}$$

$$\widetilde{E}(t) = b_E^{(1)} + b_E^{(2)} \cos\left[2\pi\left(t + b_E^{(3)}\right)/D\right] \widetilde{E}_2(t) + \epsilon_{E,2}$$
$$\widetilde{G}(t) = b_G^{(1)} + b_G^{(2)} \cos\left[2\pi\left(t + b_G^{(3)}\right)/D\right] \widetilde{G}_2(t) + \epsilon_{G,2}$$

The series model

Benth and Šaltytė-Benth (2006) then modeled the two converted series by non-Gaussian Ornstein-Uhlenbeck processes, estimating parameters by one-period autoregressions. Their model took this form, with Brownian motion and pure jump Lévy process terms.

$$
\begin{aligned}
\mathrm{d}\widetilde{E}(t) &= -\alpha_E \widetilde{E}(t)\,\mathrm{d}t + \sigma_E\,\mathrm{d}B_E(t) + \mathrm{d}L_E(t) \\
\mathrm{d}\widetilde{G}(t) &= -\alpha_G \widetilde{G}(t)\,\mathrm{d}t + \sigma_G\,\mathrm{d}B_G(t) + \mathrm{d}L_G(t)
\end{aligned}
$$

The series of residuals for electricity and gas from their study are what we use here as our fundamental data.

The series model

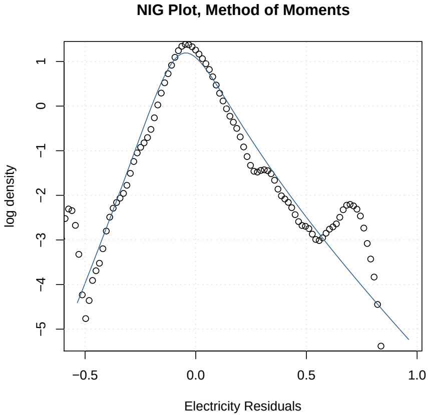
Benth and Šaltytė-Benth (2006) then modeled the two converted series by non-Gaussian Ornstein-Uhlenbeck processes, estimating parameters by one-period autoregressions. Their model took this form, with Brownian motion and pure jump Lévy process terms.

$$d\widetilde{E}(t) = -\alpha_E \widetilde{E}(t)\,dt + \sigma_E\,dB_E(t) + dL_E(t)$$
$$d\widetilde{G}(t) = -\alpha_G \widetilde{G}(t)\,dt + \sigma_G\,dB_G(t) + dL_G(t)$$

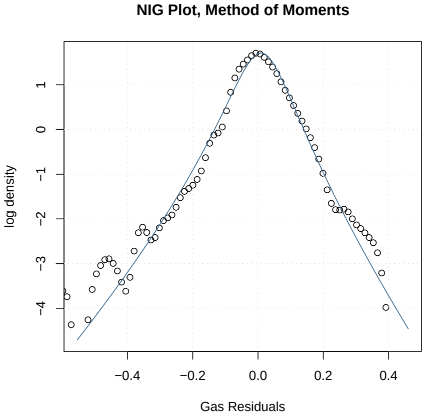
The series of residuals for electricity and gas from their study are what we use here as our fundamental data.

In this study we estimate parameters of the normal inverse Gaussian distribution by the method of moments. The following two charts are graphic depictions of the fits of these distributions.

Electricity residuals, NIG fit



Gas residuals, NIG fit



Observing and formulating

Traditionally investigators have made assumptions about the variables, for example,

- that they are normally distributed,

Observing and formulating

Traditionally investigators have made assumptions about the variables, for example,

- that they are normally distributed,
- and that their dependency is revealed by correlation.

Observing and formulating

Traditionally investigators have made assumptions about the variables, for example,

- that they are normally distributed,
- and that their dependency is revealed by correlation.

Herein we assumed a more general framework, built around the concept of a copula.

The joint distribution

From this point we considered the two time series as successive draws from a bivariate probability distribution. As such, we computed an empirical copula, and then proposed a theoretical model for it. Recall that a copula $C(v,z) : [0,1]^2 \to [0,1]$ is a function based on a bivariate distribution function.

$$\begin{aligned} F(x,y) &: \mathbb{R}^2 \to [0,1], \text{ with marginal distributions} \\ F_1(x) &: \mathbb{R} \ \to [0,1], \text{ and} \\ F_2(y) &: \mathbb{R} \ \to [0,1], \text{ such that} \end{aligned}$$

$$C\big(F_1(x), F_2(y)\big) = F(x,y),$$

$$\text{or equivalently,} \quad C(v,z) = F\big(F_1^{-1}(v), F_2^{-1}(z)\big)$$

The copula difference

For analysis it may be more convenient, and is in this case, to look at the difference between a copula and the independent copula

$$C_{\perp}(v,z) = vz,$$

which represents all independent marginal distributions.

This difference we define as follows.

$$C_{\Delta}(v,z) := C(v,z) - C_{\perp}(v,z)$$

Specification of the copula model

After studying the data we proposed the following model, which captures the triangular and parabolic nature of the data, while separating the variables.

$$\widehat{C}_{\Delta}(v,z) = h(1-|2v-1|)[1-(2z-1)^2],$$

where h is a parameter to be estimated.

Specification of the copula model

After studying the data we proposed the following model, which captures the triangular and parabolic nature of the data, while separating the variables.

$$\widehat{C}_{\Delta}(v,z) = h(1 - |2v - 1|)[1 - (2z - 1)^2],$$

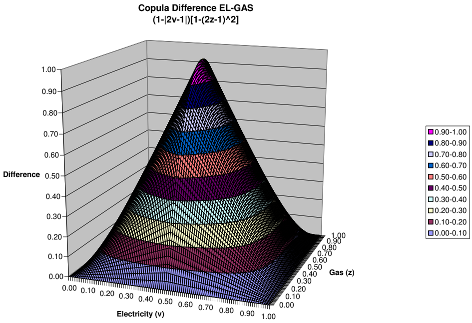
where h is a parameter to be estimated.

As estimated, $h = 0.0848$, which places the copula $C(v,z)$ almost exactly 2/3 of the distance on a linear scale between the limiting Fréchet lower and upper limit copulas.

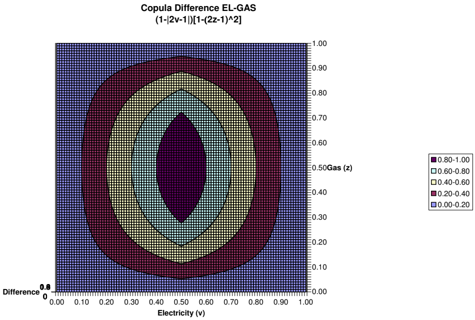
Views of the model copula difference and level curves

Next is a perspective view of the model $\widehat{C}_{\Delta}(v, z)$, with $h = 1$ before scaling to fit the data by least squares. After that chart is a vertical view showing the level curves.

Copula Difference, unscaled



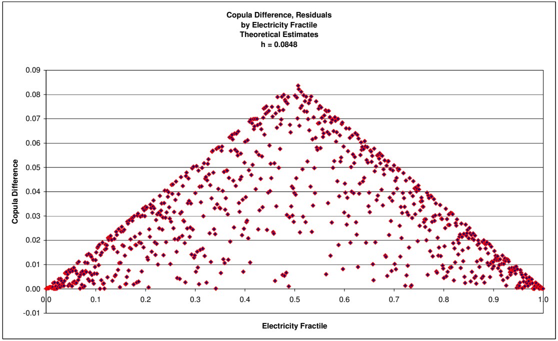
Copula Difference, Level Curves



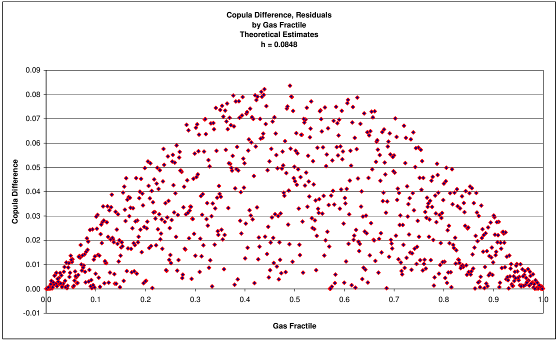
Graphics of the estimated copula differences

Now coming are two projection charts of the estimated copula difference $\widehat{C}_{\Delta}(v, z)$ as calculated for the same domain points as the empirical copula.

Copula Difference Electricity, Estimate



Copula Difference Gas, Estimate



Modelling and inferring

Now we are ready to construct a model and to formulate conclusions.

- We propose an Ornstein-Uhlenbeck model based on our copula.

Modelling and inferring

Now we are ready to construct a model and to formulate conclusions.

- We propose an Ornstein-Uhlenbeck model based on our copula.
- We see that the simulated results conform well to the data.

Modelling and inferring

Now we are ready to construct a model and to formulate conclusions.

- We propose an Ornstein-Uhlenbeck model based on our copula.
- We see that the simulated results conform well to the data.

The results are satisfying in view of the relative simplicity of the model.

Draws from the theoretical copula with NIG marginals

We constructed simulated paths for the residuals by first drawing from the theoretical copula starting with two uniform variates (the second conditional on the first,) and inverting through the fitted NIG marginal distributions.

Draws from the theoretical copula with NIG marginals

We constructed simulated paths for the residuals by first drawing from the theoretical copula starting with two uniform variates (the second conditional on the first,) and inverting through the fitted NIG marginal distributions.

Then, we made the simulated steps by the hypothesized Ornstein-Uhlenbeck processes in the variables, as reported by Benth and Šaltytė-Benth (2006). We used herein the estimates from their models.

Draws from the theoretical copula with NIG marginals

We constructed simulated paths for the residuals by first drawing from the theoretical copula starting with two uniform variates (the second conditional on the first,) and inverting through the fitted NIG marginal distributions.

Then, we made the simulated steps by the hypothesized Ornstein-Uhlenbeck processes in the variables, as reported by Benth and Šaltytė-Benth (2006). We used herein the estimates from their models.

For control purposes we employed the same uniform draws to construct bivariate normal paths using the means and covariance of the empirical sample.

Draws from the theoretical copula with NIG marginals

We constructed simulated paths for the residuals by first drawing from the theoretical copula starting with two uniform variates (the second conditional on the first,) and inverting through the fitted NIG marginal distributions.

Then, we made the simulated steps by the hypothesized Ornstein-Uhlenbeck processes in the variables, as reported by Benth and Šaltytė-Benth (2006). We used herein the estimates from their models.

For control purposes we employed the same uniform draws to construct bivariate normal paths using the means and covariance of the empirical sample.

Finally, we prepared 1000 bivariate paths of length 20 for each mode of draw – NIG and binormal.

Steps of the simulation

Following the Ornstein-Uhlenbeck model, here are the steps to generate paths, where the error terms are the residuals replacing the Brownian motion and pure jump Lévy process terms.

$$x_E(1) = \mu_E + \epsilon_E(1)$$
$$x_E(i) = (1 - \alpha_E)\, x_E(i-1) + \epsilon_E(i), \quad 1 < i \leqslant p$$

$$x_G(1) = \mu_G + \epsilon_G(1)$$
$$x_G(i) = (1 - \alpha_G)\, x_G(i-1) + \epsilon_G(i), \quad 1 < i \leqslant p$$

Simulated spark spread series

In preparation for the option evaluation study we generated the price series for spark spreads for four 20-day periods evenly spaced about a year.

- We took the terminal simulated path points in the NIG and binormal evaluations and reversed the process by which the residuals were calculated by Benth and Šaltytė-Benth (2006).

Simulated spark spread series

In preparation for the option evaluation study we generated the price series for spark spreads for four 20-day periods evenly spaced about a year.

- We took the terminal simulated path points in the NIG and binormal evaluations and reversed the process by which the residuals were calculated by Benth and Šaltytė-Benth (2006).
- This process amounted to reseasonalizing, retrending, exponentiating, and forming the spread differences, making allowance for the effect of the heat rate unit conversion.

Simulated spark spread series

In preparation for the option evaluation study we generated the price series for spark spreads for four 20-day periods evenly spaced about a year.

- We took the terminal simulated path points in the NIG and binormal evaluations and reversed the process by which the residuals were calculated by Benth and Šaltytė-Benth (2006).
- This process amounted to reseasonalizing, retrending, exponentiating, and forming the spread differences, making allowance for the effect of the heat rate unit conversion.

Following these results we were ready to move forward to evaluating options.

Simulated option prices with NIG assumption

Strike	Period 1	Period 2	Period 3	Period 4	Type
-10	8.7248	12.4448	14.5029	11.3708	Call
-10	0.0107	0.0013	0.0011	0.0105	Put
0	0.6360	2.5609	4.5424	2.0509	Call
0	1.9220	0.1175	0.0406	0.6906	Put
10	0.0233	0.0442	0.1746	0.1134	Call
10	11.3092	7.6008	5.6728	8.7531	Put
mean	-1.2860	2.4434	4.5018	1.3603	—
dev.	3.0141	2.5945	3.1629	3.8034	—

Simulated option prices with binormal assumption

Strike	Period 1	Period 2	Period 3	Period 4	Type
-10	8.7681	12.4869	14.5545	11.4260	Call
-10	0.0033	0.0000	0.0000	0.0017	Put
0	0.7152	2.6820	4.6235	2.3058	Call
0	1.9504	0.1950	0.0690	0.8815	Put
10	0.0008	0.0075	0.0921	0.0384	Call
10	11.2359	7.5206	5.5376	8.6140	Put
mean	-1.2352	2.4869	4.5546	1.4243	—
dev.	3.0728	2.6160	3.1761	3.8535	—

For additional reading…

- Benth, F. E. and J. Šaltytė-Benth (2004).
  The normal inverse Gaussian distribution and spot price modelling in energy markets.
  Int. J. Theoretical Appl. Finance 7(2), 177–192.
- Benth, F. E. and J. Šaltytė-Benth (2006).
  Analytical approximation for the price dynamics of spark spread options.
  Stud. Nonlinear Dynam. Econometrics.
  To appear in special issue, "Nonlinear analysis of electricity prices".
- Cherubini, U., E. Luciano, and W. Vecchiato (2004).
  Copula Methods in Finance.
  Chichester: Wiley.

To reach me —

"Paul C. Kettler" <paulck@math.uio.no>

www.paulcarlislekettler.net

Telephone: +47 2285 7771