Course Documents

1 Final Exam, with Answers

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FinalExam03_Answers.docx (18.823 Kb)

Here attached is a copy of the Final Exam, with Answers.

2 Cox and Rubinstein pp. 204–235

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Cox and Rubinstein pp. 204–235 (889.508 Kb)

Here attached is a scan of pp. 204–235 in the Cox and Rubinstein book, reference below. Start reading at, "Convergence to the Black–Scholes Option Pricing Formula," on p. 204. Included is an analysis showing that the multi-period binomial pricing model converges to the continuous Black–Scholes model in the limit as the partition of the time line becomes increasingly refined. Note that there is a difference between any multi-period binomial model, however refined, and the continuous model. Every path of the binomial model has finite [first order] length, meaning the sum of the absolute values of the jumps, whereas almost every path (except those forming a set of measure zero) of the continuous model has infinite first order length. The continuous model, incorporating Browning motion, does however have 'quadratic variation,' meaning that the squares of the infinitesimal changes add to a finite number, in fact to the length of time elapsed. This fact is incorporated in the fundamental differential equation of Brownian motion, $dB_t^2 = dt$.

I suggest you save this file locally, then open it with Adobe Reader. You will then need to go to the View Menu → Rotate View → Counter Clockwise, to see the pages properly oriented. From there you can resize the image to your convenience.

Cox, John C. and Mark Rubinstein, Options Markets, Prentice–Hall, Upper Saddle River, New Jersey, 1985.

3 Mean and variance of terminal stock price in one-period option pricing model

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StockPriceStats.docx (12.437 Kb)

See the document at this link.

4 Discussion of Slides 5, 7, and 10 for Chapter 9

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Slide 5: The setup: *Suppose that the price of a call is 3, while the stock is 20, the strike 18, with one period to expiration, interest at 10%, no dividends. Is there an arbitrage opportunity?*

Think parity. There is an arbitrage because the call at 3.00 is less than its intrinsic value, the difference between the stock at 20.00 and the discounted strike at 16.20, if positive, which in this case it is. No matter what the price of the put, therefore (the higher, the better,) the strong>reversal strategy is a winner. Go long the call, short the put, and short the stock, investing the present value of the strike price, plus the excess. The return to maturity is better than the risk-free rate.

Slide 7: The setup: *Suppose that the price of a put is 1, while the stock is 37, the strike 40, with one-half period to expiration, interest at 5%, no dividends. Is there an arbitrage opportunity?*

Think parity. There is an arbitrage because the put at 1.00 is less than its intrinsic value, the difference between the discounted strike at 39.00 and the stock at 37.00, if positive, which in this case it is. No matter what the price of the call, therefore (the higher, the better,) the conversion strategy is a winner. Go long the put, short the call, and long the stock, borrowing the present value of the strike price, less the deficiency. The return to maturity is better than the risk-free rate.

Slide 10: The setup: *Suppose that the price of a call is 3, while the stock is 31, the strike 30, with one-quarter period to expiration, interest at 10%, no dividends. Is there an arbitrage opportunity if the put is at 2.25? if the put is at 1.00?*

Consider parity once again.The difference of the stock at 31.00 and the present value of the strike at 29.25 is 1.75. In the first case the difference of the call at 3.00 and the put at 2.25 is 0.75. Therefore the conversion is overpriced. Do the reversal. In the second case the difference of the call at 3.00 and the put at 1.00 is 2.00. Therefore the conversion is underpriced. Do it.

5 Answers to Hull text Problems 9.23 and 9.24 Modify Manage Copy Remove

Problem 9.23. The position entered is known as the "call butterfly." To begin, it is useful to determine the value of this position at expiration. Such value is a function of the stock price S_T at the terminal time T. One only need to determine the pattern of exercise for the various calls at this time to assess value. If $S_T \le K_1$, then no option will be exercised, and therefore there will be no cash flows, so the butterfly has value zero. If $K_1 \le S_T \le K_2$, then c_1 will be exercised, but no other, resulting in the [positive] cash flow of $S_T - K_1$. If $K_2 \le S_T \le K_3$, then c_1 will be exercised, and also the two calls c_2, resulting in the [positive] cash flow of $(K_2 - K_1) - (S_T - K_2) \ge 0$. If $K_3 \le S_T$ then all four calls will be exercised, resulting in a zero cash flow, and therefore a zero value for the butterfly. Insofar as all cash flows are non-negative in the four domains, the butterfly cannot have a negative value at expiration, nor can it have a negative value at any earlier time, regardless of the probability measure assigned to the terminal stock value S_T. In the more general case of American options, early exercise by the party long the two calls c_2 only brings the terminal value of the butterfly closer, and therefore enhances value.

The function of butterfly value at expiration is therefore a triangular function with maximum of $(K_2 - K_1) = (K_3 - K_2)$ [by hypothesis] when $S_T = K_2$. Comparing the value of $2c_2$ with the sum of calls $c_1 + c_3$ demonstrates that $2c_2 = c_1 + c_3$ in the extreme domains, and $2c_2 \le c_1 + c_3$ when $K_1 \le S_T \le K_3$, proving the assertion.

Problem 9.24. The result obtains *mutatis mutandis* following the discussion for calls.

6 Midterm Examination, with Answers

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Midterm exam02_Answers.docx (20.268 Kb)

Here are solutions to the problems on the Midterm Exam. Owing to some misleading specifications of Problem 4, all students will receive full credit for it. A revised problem, with corrections, appears in this document. I regret these errors, and will use the revised problem as a learning tool for all, including me.

7 Midterm Exam Preparation

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Hi all --

Several students have asked me for direction to study for the midterm examination. Here it is.

The material to be covered includes only those concepts which are presented in substantial form both in the lectures and the book. This means the intersection of the two, not the union. Thus, if you have studied your lecture notes only, you will be prepared. And if you study the book for the assigned readings, slides, and problems, you will also be prepared. Naturally, since I am giving the lectures and also making the exam you can reasonably expect that the problems on the exam will look much like the examples I put on the board.

As advertised, this is an 'open book' examination, so you may bring with you any written materials you desire, even a hand held calculator, even one that accesses the Internet. You may be assured, however, that such calculator will provide no advantage. Any numbers to be calculated will be easy to find using pencil and paper. This is not a course in doing arithmetic, so I will not subtract points for simple errors in calculation. The setup of a solution will be the most important part, and a correct setup, with proper formulation, will receive full credit.

I ask you not to bring laptop, notebook, or tablet computers, simply because they can provide distractions to those working nearby.

The exam will have no more than six questions, and will be a "thinking person's" exam, meaning that application of principles will be paramount. Recall of reference material will play no part, though you may help focus yourself on a correct solution by looking to your course notes or book. You can be certain, though, that the answers will not be there, or on the Internet.

Using my experience as a guide you will assess my exam as fair, mainstream without tricks, and a useful learning experience. I wish you all good luck. Incidentally, I do not grade on a 'curve', except in the large sense of what all people similarly situated to you can expect of yourselves. Thus is is possible, but unlikely, that all will receive perfect scores. I fully understand that students have different priorities in their academic programs, and that those placing more emphasis on this course will likely do better. In my mind you are all capable of the best results, given sufficient application of effort.

Your humble servant,

Paul

8 Hull Slides

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Hull Slides.zip (32.989 Mb)

9 Syllabus

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Syllabus.pdf (29.277 Kb)

Here is the Syllabus for the course.

OK